Accurate atmospheric correction of two-frequency SLR observations

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Abstract

We have developed a new atmospheric correction formula for two-frequency SLR observations based on the theory of the two-frequency range correction. This new formula eliminates the total atmospheric density effects and takes into account all the remaining propagation effects except those caused by atmospheric turbulence. Numerical simulations show that this new formula completely reduces all propagation effects at any elevation angle with an accuracy better than 1 mm.
The required information about the water vapour distribution along the propagation path can be calculated using GPS or Water Vapour Radiometer data. The accuracy demand on this data is moderate, thus we propose to use a co-located GPS receiver. The curvature effects can be calculated by an accurate ray-tracing algorithm or a model. However, the required precision for the difference of the two-frequency SLR measurements, i.e. better than 30µm for a single epoch, exceeds the capability of the current state-of-the-art SLR systems.

1. Motivation

The International Association of Geodesy (IAG) has inagurated a new global service, namely, Global Geodetic Observing System (GGOS). This global service is dedicated to ensure precise long-term monitoring of the geodetic observables related to the global reference frame definition, the dynamic of atmosphere and ocean, the global hydrological cycle as well as natural hazards and disasters by integrating different geodetic techniques, different models and different approaches (see www.ggos.org). In the frame of GGOS, the accuracy requirement of Satellite Laser Ranging (SLR) observations has to be at the millimeter level (Rothacher, 2005). However, the demand to increase the accuracy of the measurements has shown that their ultimate accuracy is limited by the correction of the atmospheric propagation effects.

For the single frequency SLR system, Marini and Murray (1973) proposed in 1973 their model reflecting the precision requirement at that time. Since then all known developments of improved atmospheric correction formulae were based on the Marini-Murray scheme. For example, Mendes et al. (2002) developed new mapping functions (FCULa and FCULb) to scale the zenith delay to other elevation angles. Finally, Mendes and Pavlis (2004) proposed a new zenith delay model which was then adopted as the standard model for refraction modeling. The latest progress is due to Hulley and Pavlis (2007) who used a ray-tracing technique to calculate the propagation effects, including the effects of horizontal refractivity gradients.

The alternative to modeling is the use of two-frequency SLR measurements for the direct computation of the propagation effects by utilizing the dispersion in the electrically neutral atmosphere. The dispersion causes the optical path lengths (OPLs) at two different frequencies to differ in proportion to the path integrated atmospheric density. Thus, the difference of the two OPLs can be used to calculate one of the propagation effects. This system obviously has the potential to improve the accuracy of SLR results. For a two-frequency SLR system, one atmospheric correction formula is known that originally was proposed by M.T. Prilepin (see ref [12] in Abshire and Gardner, 1985) and his formula was later studied by several investigators (e.g. Bender and Owens,1965; Abshire and Gardner, 1985; Greene and Herring, 1986). It has become the standard atmospheric correction formula for two-frequency SLR systems.

The standard formula can reduce the largest part of the propagation effects, namely the dry atmospheric density. The remaining effects such as the water vapor density and curvature effects are neglected. At the optical frequencies, water vapor contributes only about 1% of the group refractivity, however, it can introduce substantial errors. Numerical simulations show that the water vapor effects amount to a few mm for SLR observations in the zenith direction. Furthermore, the magnitude of the curvature effects could be a few cm for observations taken at an elevation angle lower than 10^0. Based on the above facts, it seems important to improve the standard formula by considering all propagation effects, so that the accuracy of SLR observations as required by the GGOS service can be fulfilled.

We have developed a new atmospheric correction formula for two-frequency SLR observations based on the theory of the two-frequency range correction of Gu and Brunner (1990). The propagation effects of the first and second frequency measurements are evaluated along the same ray path. This new formula eliminates the total atmospheric density effect including its gradient and provides two terms to calculate the water vapor and curvature effects. In the following sections, we first present briefly the derivations of the new formula. Then we summarize results from the investigations of the new formula.

2. Derivation of the new formula

Since the electrically neutral atmosphere is a dispersive medium for optical waves, the propagations of the first and second SLR frequencies are slightly different. In Figure 1, we depict a slight separation of the ray path p_1 of frequency f_1 from the ray path p_2 of frequency f_2, where $f_1 < f_2$ is assumed. Based on the theory of the two-frequency range correction, the propagation effects of frequency f_2 can be evaluated along the ray path p_1,or vice versa. Thus, we are able to investigate the propagation effects of both frequencies along the same ray path. Following the assumptions made by Gu and Brunner (1990), the derivations presented here also assume the applicability of the geometrical optics approximation and neglect atmospheric turbulence effects.

The OPLs for f_1 and f_2 are denoted by R_1 and R_2, respectively. The straight line distance between the two points O and X is denoted by S. For a specific time instance, the refractive index n can be expressed as a function of frequency f_1 and position $\vec{r}_i$, $n(f_i, \vec{r}_i)$. The OPL is obtained by integrating the refractive index along the ray path that satisfies the ray equation (Born and Wolf, 1999) and can then be expressed as

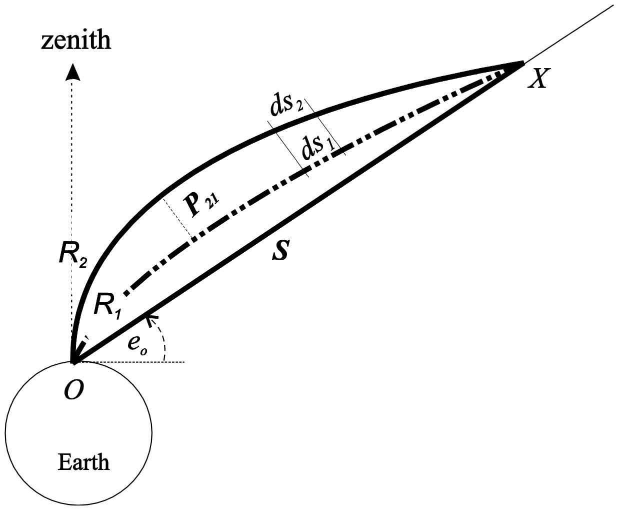

Figure 1. The geometrical scheme of the propagation of SLR signals through the electrically neutral atmosphere. The OPL for f_1 (R_1) is slightly different to that for f_2 (R_2). P_{21} is the propagation correction term and e_o is the chord elevation angle.

$$R_1 = \int_{p_1} n(f_1, \vec{r}_1) ds_1 \tag{1}$$

$$R_2 = \int_{p_2} n(f_2, \vec{r}_2) ds_2 \tag{2}$$

In order to use R_2 for the calculation of the propagation effects, the integral in Eq.(2) needs to be expressed along the ray path p_1 rather than p_2. Thus, following Gu and Brunner(1990), R_2 may be expressed as

$$R_2 = \int_{p_1} n(f_2, \vec{r}_1) ds_1 + P_{21} \tag{3}$$

where the term P_{21} represents the propagation corrections from the ray path p_1 to p_2 and is expressed as

$$P_{21} = \int_{p_2} n(f_2, \vec{r}_2) ds_2 - \int_{p_1} n(f_1, \vec{r}_1) ds_1 \tag{4}$$

Furthermore, we can express $S = \int_{p_1} ds_1 - K_1$ and by combining it with Eq.(1) and Eq.(3), we obtain

$$R_1 = S + \int_{p_1} [n(f_1, \vec{r}_1) - 1] ds_1 + K_1 \tag{5}$$

$$R_2 = S + \int_{p_1} [n(f_2, \vec{r}_1) - 1] ds_1 + K_1 + P_{21} \tag{6}$$

K_1 is the arc-to-chord correction for the ray path p_1. General expression for K_1 and P_{21} can be found in Gu and Brunner (1990).

The OPLs R_1 and R_2 are now expressed along the same ray path, which allows us to calculate the propagation effects along the ray path p_1 only. The next step is to provide an expression for the refractivity term $[n(f_i,\vec{r}_i)-1]$. A separation of a frequency term from the atmospheric variables would be very useful. Thus, we express the refractivity as a function of the atmospheric density ρ, which depends on time and position, and the dispersion constant k, which depends on frequency only. The refractivity can be expressed as

$$10^6\left[n(f_i,\vec{r}_i)-1\right]=k_d(f_i)\,\rho_t(\vec{r}_i)+k_v^*(f_i)\,\rho_v(\vec{r}_i) \tag{7}$$

where $k_v^*(f_i)=k_v(f_i)-k_d(f_i)$. $k_d(f_i)$ and $k_v(f_i)$ are the dispersion constants for dry air and water vapor, respectively. ρ_t and ρ_v are the atmospheric total density and the water vapor density, respectively. By substituting the above equation into Eq.(5) and Eq.(6), we obtain

$$R_1=S+10^{-6}k_d(f_1)\int_{p_1}\rho_t(\vec{r}_1)ds_1+10^{-6}k_v^*(f_1)\int_{p_1}\rho_v(\vec{r}_1)ds_1+K_1 \tag{8}$$

$$R_2=S+10^{-6}k_d(f_2)\int_{p_1}\rho_t(\vec{r}_1)ds_1+10^{-6}k_v^*(f_2)\int_{p_1}\rho_v(\vec{r}_1)ds_1+K_1+P_{21} \tag{9}$$

From the above equation, it is clear the OPLs R_1 and R_2 contain the same quantities of the atmospheric total density and the water vapor density as well as the curvature effects, which are evaluated along the same ray path p_1. Therefore, we can rigorously eliminate the unknown integral $\int_{p_1}\rho_t(\vec{r}_1)ds_1$, which now yields

$$S=R_1+\nu(R_1-R_2)+(\nu P_{21}-K_1)+\mathrm{H}_{21}.\mathrm{SIWV} \tag{10}$$

where the power of dispersion ν is

$$\nu=\frac{k_d(f_1)}{k_d(f_2)-k_d(f_1)} \tag{11}$$

the water vapor factor H_{21} is

$$\mathrm{H}_{21}=10^{-6}k_v^*(f_1)\,\nu\left(\frac{k_v^*(f_2)}{k_v^*(f_1)}-\frac{k_d(f_2)}{k_d(f_1)}\right) \tag{12}$$

the slant integrated water vapor (SIWV) is

$$\mathrm{SIWV}=\int_{p_1}\rho_v(\vec{r}_1)ds_1 \tag{13}$$

The second term in Eq.(10) contains the dispersion effects, the third term represents the curvature effects of the ray path p_1 and the propagation corrections from p_2 to p_1, and the last term represents effects of the water vapor density. Furthermore, the second term can be obtained from the observed $(R_1 - R_2)$ data. The terms K_1 and P_{21} can be calculated using a ray-tracing technique or a model. SIWV can be observed by using an external technique such as GNSS or Water Vapor Radiometer (WVR). The constants ν and H_{21} can be easily calculated using the dispersion formulae.

3. Evaluation and investigation

3.1 Numerical simulation

We have investigated the values of all terms in Eq.(10) using a 2D ray-tracing technique. The frequencies used in the numerical simulations are those of the two-frequency SLR system of the Graz station, λ_1 = 532 nm and λ_2 = 1068.4 nm (Kirchner, *personal communication*). Radiosonde data from World Meteorological Organization (WMO) during 2007 to 2008 are used in the calculations. The refractivity values are calculated based on the Ciddor (1996) procedures. The mean values of all correction terms are presented in Table 1. The constant values of ν and H_{21} are -22.2065 and $1.35 \, x \, 10^{-4}$ m^3 kg^{-1}, respectively.

The mean value of the term $(R_1 - R_2)$ is about -2.3275 m for observations in the zenith direction and it increases to -33.8711 m for observations down to 3^0 elevation angle. The mean values of the term $(\nu P_{21} - K_1)$ can be as large as 0.35 m at 3^0 elevation angle and its values become insignificant at elevation angles above 30^0. Furthermore, the mean values of term $(H_{21} . SIWV)$ is in the order of few mm to cm for all elevation angles.

Finally, to evaluate the performance of the new formula, we calculated the residual range error (RRE) that is defined as RRE $\equiv S - S_{RT}$. The term S_{RT} is the chord distance S that is calculated using the ray tracing technique. The mean values of RRE are listed in the last column of Table 1 (unit in nm) and indicate that the new formula can reduce all propagation effects at any elevation angle with an accuracy better than 1 mm.

Table 1. The 2-year mean and standard deviation of all correction terms and RRE

e_o (°)	$(R_1 - R_2)$ (m)	$(\nu P_{21} - K_1)$ (mm)	$H_{21} . SIWV$ (mm)	RRE (nm)
3	-33.8711 ± 0.2820	-351.84 ± 14.83	36.49 ± 17.54	-2.5
5	-23.5023 ± 0.1909	-128.55 ± 4.66	23.62 ± 11.39	-0.1
10	-12.9089 ± 0.1038	-22.51 ± 0.70	12.34 ± 5.96	7.8
15	-8.8409 ± 0.0709	-7.31 ± 0.23	8.35 ± 4.04	2.8
20	-6.7416± 0.0541	-3.04 ± 0.09	6.34 ± 3.06	-9.2
30	-4.6375 ± 0.0372	-0.84 ± 0.03	4.35 ± 2.10	3.7
40	-3.6145± 0.0290	< -0.3	3.38 ± 1.64	-0.2
60	-2.6864 ± 0.0215		2.51 ± 1.21	-4.9
90	-2.3275 ± 0.0187		2.18 ± 1.05	-3.6

3.2 Precision requirement

In order to calculate S with the 1 mm level of precision, we need to investigate precision requirements of all terms in Eq.(10). We have carried out the precision analysis using the variance propagation law with the assumption that the OPLs R_1 and R_2 are observed simultaneously. The results are briefly presented here.

An error of 1 kg m^{-2} of $SIWV$ introduces an error of about 0.2 mm to S. Thus, the precision requirement of $SIWV$ is moderate, which must only be better than 5 kg m^{-2}. The term SIWV can be calculated from a quantity SWD (slant wet delay). If this quantity is used to correct for the water vapor effect, then its precision must be better than 4 cm. This SWD requirement may be achieved by using GNSS or WVR technique.

The curvature term K_1 needs to be calculated with 1 mm precision. It was already mentioned that the magnitude of this term is only significant at low elevation angles. This term may be calculated using an accurate ray-tracing technique. Alternatively, a model could be derived for this term, however, the model's uncertainty has to be carefully considered.

The term $(R_1 - R_2)$ has to be observed with a precision better than 30μm for a single measurement epoch. This high requirement is due to the amplification by a large constant ν, which can be in the order of 10 to 200. Nowadays, modern kHz SLR systems can produce a precision of normal point data at the level of 0.37 mm (Hamal et al., 2005). Thus, the precision demand for this term exceeds the capability of the current state-of-the-art SLR systems. Similarly, the term P_{21} has to be also calculated with a precision better than 30 μm.

The precision of k_d and k_v, which are used to calculate the constant ν and H_{21}, must be better than 10^{-6}. According to Ciddor (1996), the uncertainties of the dispersion formulae proposed by some investigators (e.g. Edlen,1996; Owens,1967; Ciddor, 1996) are in the order of 10^{-6} to 10^{-9}. We recommend to use the formula developed by Ciddor (1996).

4. Concluding remarks

For the calculation of water vapor effects, we propose to use a co-located GPS receiver as it is readily available at all SLR stations and it also can produce the SWD values with a higher accuracy than required. There are two possible co-location scenarios: (i) co-located SLR and GPS observations to a satellite equipped with a retroreflector, and (ii) co-located SLR and GPS observations to different satellites. For the first scenario, the SWD can be calculated from a single GPS signal (e.g. GPS36 or GPS35). The propagation paths of SLR and GNSS signals deviate only slightly (optical and microwave paths) and hence they are assumed to carry the same information about the water vapor distribution. For the second scenario, the SWD values along the propagation paths of the SLR signal can be calculated by interpolating the SWD values obtained from processing the GPS data. It is important to mention that, the SWD observations by GPS include also the horizontal gradient of the SWD.

Since the current SLR systems can not fulfill the precision demand of the term $(R_1 - R_2)$, we are studying the application of averaging techniques to improve the precision. Using the new formula the bias (propagation effects) can be removed but the noise is amplified. The noise, however, can be reduced by averaging techniques. This, of course, requires a careful study of the turbulence effects.

The new formula developed in this paper improves the standard formula by adding two additional terms to calculate the curvature and water vapor effects. In particular, we thoroughly consider the propagation paths of f_1 and f_2 before eliminating one of the unknown terms. In this case, we take into account the propagation term P_{21} that allows us to evaluate the propagation effects of frequency f_2 along the ray path of frequency f_1. This makes the rigorous elimination of the term $\int_{p_1} \rho_t(\vec{r}_1)ds_1$ possible. Finally, we would like to emphasis that the new formula is derived based on the assumption of simultaneous R_1 and R_2 observations. Only for this situation can the instantaneous value of $\int_{p_1} \rho_t(\vec{r}_1)ds_1$ be eliminated accurately.

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